STIGATION OF SHEAR LAG IN PANELS WITH AND WITHOUT CUTOUTS

by

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May 1950

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ACKNOWLEDGEMENT

It is the desire of the writers of this paper to express acknowledgement to Assistant Professor Raymond I. Schneyer of the Aeronautical Engineering Department of the University of Michigan for his guidance and advice throughout the period of this investigation.

It is further desired to express appreciation to Mr. William Spitza of the Aeronautical Engineering Department machine shop of the University of Michigan for his excellent work in the preparation and mounting of the test specimens and other apparatus used in the experimental portion of this investigation.

TABLE OF CONTENTS

SUMMARY

At the suggestion of Assistant Professor Raymond I. Schneyer, of the Department of Aeronautical Engineering, University of Michigan, Ann Arbor, Michigan, the writers, Commander Richard H. Lachman and Lt. Commander Vance A. Schweitzer, USN, undertook an investigation of shear lag in panels with and without cutouts in order to seek experimental verification of a new theoretical method of solution proposed by Professor Schneyer. This investigation was carried out as a partial fulfillment of requirements for the degree of Master of Science in Aeronautical Engineering.

Three experimental models were used in this investigation: the first, a three-stringer unsymmetrical model with no cutouts; the second, a four-stringer symmetrical model with no cutouts; and the third, a four-stringer symmetrical model having a cutout in one of the panels. All models were constructed of 24-ST aluminum alloy.

The results of this investigation indicated that Professor Schneyer's proposed method of solution gave a completely accurate analysis of the shear lag problem in the models investigated, commensurate with the degree of accuracy of experimental measurements obtained.

INTRODUCTION

This investigation of the effect of shear lag in panels with and without cutouts was undertaken by the writers as a thesis project for partial fulfillment of requirements for the degree of Master of Science in Aeronautical Engineering from the Horace H. Rackham School of Graduate Studies, University of Michigan, Ann Arbor, Michigan.

The subject of this investigation was suggested by Assistant Professor Raymond I. Schneyer of the Aeronautical Engineering Department of the University of Michigan. Professor Schneyer has devoted much effort in developing a new theoretical approach to the problem of transmission of shear about cutouts. It is the primary intent of this paper to seek experimental verification of the results predicted by Professor Schneyer's method.

Previous theoretical studies on this subject have been conducted by other individuals in the field, including Mr. Paul Kuhn, whose paper, reference (a), provided the basis for Professor Schneyer's extended theory.

The basic elements of Professor Schneyer's theory are presented in a subsequent section of this paper. A complete development of this theory is to be presented by Professor Schneyer in a paper to be published in the near future.

LIST OF SYMBOLS

u - Total deformation of Stringer

σ - Stresses in Stringer

A - Area of Stringer

τ - Shear Stress in Sheet

P - Load in Stringer

t - Thickness of Sheet

E - Modulus of Elasticity

G - Shear Modulus

λ - Panel Width

L - Length of Panel

BASIC THEORY OF THE SHEAR LAG PROBLEM

The basic theory of shear lag is developed for a three-stringer panel having stiffeners of constant cross-section, but with no restrictions placed on sheet thicknesses nor symmetrical dimensions of configuration. Only the end results of this development are presented herein.

The assumptions utilized in this development are as follows:

(1) Changes in dimensions, under load, take place only in the longitudinal direction. Any changes occurring in the lateral direction are considered to be of minor effect.

(2) The stringers carry all the tensile or compressive load applied to the structure.

(3) The plate carries only shear load. Any tension load carried by the plate can be accounted for by the use of an effective width of sheet added to the stringers.

(4) Law of superposition of loading applies.

(5) An infinite number of ribs or bulkheads exist which are rigid in their own plane, but have no rigidity in the plane of the panel.

As indicated by the procedure set forth in the section of this paper entitled "Sample Calculations", the solution of the more complicated panels is based upon the resolution of these structures into a series of simple panels for which a general solution has been obtained. Then, through the principle of superposition of loading, which has been assumed to apply, plus the use of a load distribution method, a solution of these more complicated panels is obtained. It is therefore necessary in the basic analysis of this problem to seek solutions of the three-stringer panel for a condition of loading on each of the three stringers so that the load distribution method may be applied.

The final results of these three separate solutions are given below.

Case I. Load applied at center stringer.

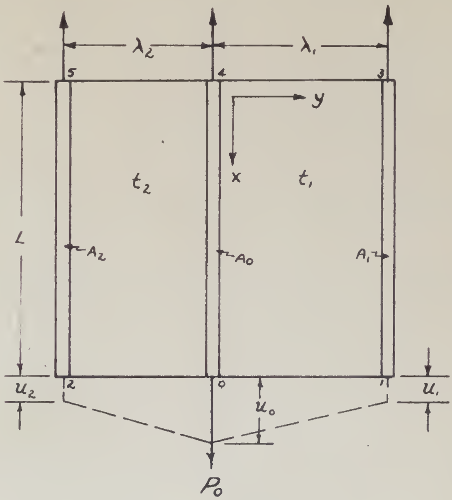


Figure 1

The solution of this problem leads to the following results:

$$\tau_1 = P_o \left(C_{01} \sinh m_1 x + C_{02} \sinh m_2 x \right)$$

$$\tau_2 = P_o \left(D_{01} \sinh m_1 x + D_{02} \sinh m_2 x \right)$$

$$\sigma_o = \frac{P_o}{A_o} + \frac{P_o}{A_o} \left[\frac{1}{m_1} \left(\cosh m_1 x - \cosh m_1 L \right) \left(t_1 C_{01} + t_2 D_{01} \right) \right.$$
$$\left. + \frac{1}{m_2} \left(\cosh m_2 x - \cosh m_2 L \right) \left(t_1 C_{02} + t_2 D_{02} \right) \right]$$

$$\sigma_1 = \frac{P_o t_1}{A_1} \left[\frac{C_{01}}{m_1} \left(\cosh m_1 L - \cosh m_1 x \right) + \frac{C_{02}}{m_2} \left(\cosh m_2 L - \cosh m_2 x \right) \right]$$

$$\sigma_2 = \frac{P_o t_2}{A_2} \left[\frac{D_{01}}{m_1} \left(\cosh m_1 L - \cosh m_1 x \right) + \frac{D_{02}}{m_2} \left(\cosh m_2 L - \cosh m_2 x \right) \right]$$

$$u_o = \frac{P_o X}{A_o E} + \frac{P_o}{A_o E} \left[\frac{1}{m_1^2} \left(\sinh m_1 x - m_1 x \cosh m_1 L \right) \left(t_1 C_{01} + t_2 D_{01} \right) \right.$$
$$\left. + \frac{1}{m_2^2} \left(\sinh m_2 x - m_2 x \cosh m_2 L \right) \left(t_1 C_{02} + t_2 D_{02} \right) \right]$$

$$u_1 = \frac{P_o t_1}{A_1 E} \left[\frac{C_{01}}{m_1^2} \left(m_1 x \cosh m_1 L - \sinh m_1 x \right) + \frac{C_{02}}{m_2^2} \left(m_2 x \cosh m_2 L - \sinh m_2 x \right) \right]$$

$$u_2 = \frac{P_o t_2}{A_2 E} \left[\frac{D_{01}}{m_1^2} \left(m_1 x \cosh m_1 L - \sinh m_1 x \right) + \frac{D_{02}}{m_2^2} \left(m_2 x \cosh m_2 L - \sinh m_2 x \right) \right]$$

Case II. Load applied on Stringer (1).

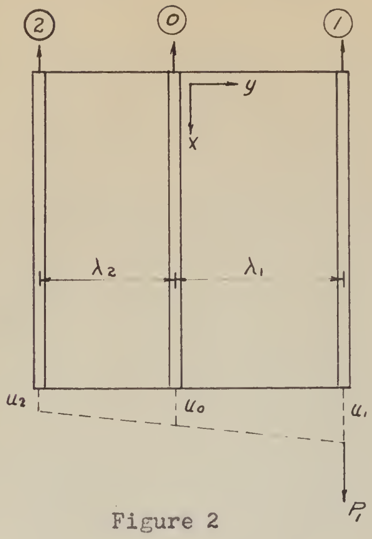


Figure 2

The solution of this problem leads to the following results:

$$\sigma_0 = \frac{P_1}{A_0}\Big[\frac{1}{m_1}(\cosh m_1 x - \cosh m_1 L)(t_1 C_{11} + t_2 D_{11})$$

$$+ \frac{1}{m_2}(\cosh m_2 x - \cosh m_2 L)(t_1 C_{12} + t_2 D_{12})\Big]$$

$$\sigma_1 = \frac{P_1}{A_1} + \frac{P_1 t_1}{A_1}\Big[\frac{C_{11}}{m_1}(\cosh m_1 L - \cosh m_1 x) + \frac{C_{12}}{m_2}(\cosh m_2 L - \cosh m_2 x)\Big]$$

$$\sigma_2 = \frac{P_1 t_2}{A_2}\Big[\frac{D_{11}}{m_1}(\cosh m_1 L - \cosh m_1 x) + \frac{D_{12}}{m_2}(\cosh m_2 L - \cosh m_2 x)\Big]$$

$$\tau_1 = P_1 (C_{11} \sinh m_1 x + C_{12} \sinh m_2 x)$$

$$\tau_2 = P_1 (D_{11} \sinh m_1 x + D_{12} \sinh m_2 x)$$

$$u_0 = \frac{P_1}{A_0 E}\Big[\frac{1}{m_1^2}(\sinh m_1 x - m_1 x \cosh m_1 L)(t_1 C_{11} + t_2 D_{11})$$

$$+ \frac{1}{m_2^2}(\sinh m_2 x - m_2 x \cosh m_2 L)(t_1 C_{12} + t_2 D_{12})$$

$$u_1 = \frac{P_1 x}{A_1 E} + \frac{P_1 t_1}{A_1 E}\left[\frac{C_{11}}{m_1^2}(m_1 x \cosh m_1 L - \sinh m_1 x) + \frac{C_{12}}{m_2^2}(m_2 x \cosh m_2 L - \sinh m_2 x)\right]$$

$$u_2 = \frac{P_1 t_2}{A_2 E}\left[\frac{D_{11}}{m_1^2}(m_1 x \cosh m_1 L - \sinh m_1 x) + \frac{D_{12}}{m_2^2}(m_2 x \cosh m_2 L - \sinh m_2 x)\right]$$

Case III. Load applied on Stringer (2).

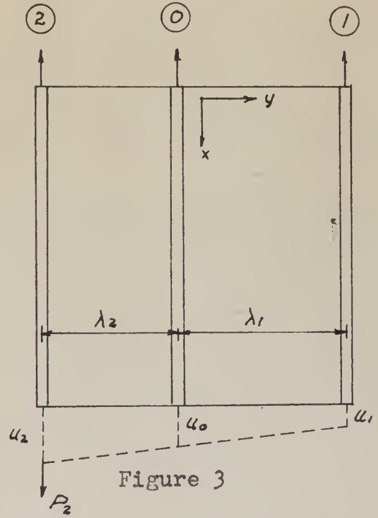


Figure 3

The solution of the problem in this case leads to the following results:

$$\tau_1 = P_2 (C_{21} \sinh m_1 x + C_{22} \sinh m_2 x)$$

$$\tau_2 = P_2 (D_{21} \sinh m_1 x + D_{22} \sinh m_2 x)$$

$$\sigma_0 = \frac{P_2}{A_0}\left[\frac{1}{m_1}(\cosh m_1 x - \cosh m_1 L)(t_1 C_{21} + t_2 D_{21}) + \frac{1}{m_2}(\cosh m_2 x - \cosh m_2 L)(t_1 C_{22} + t_2 D_{22})\right]$$

$$\sigma_1 = \frac{P_2 t_1}{A_1}\left[\frac{C_{21}}{m_1}(\cosh m_1 L - \cosh m_1 x) + \frac{C_{22}}{m_2}(\cosh m_2 L - \cosh m_2 x)\right]$$

$$\sigma_2 = \frac{P_2}{A_2} + \frac{P_2 t_2}{A_2}\left[\frac{D_{21}}{m_1}(\cosh m_1 L - \cosh m_1 x) + \frac{D_{22}}{m_2}(\cosh m_2 L - \cosh m_2 x)\right]$$

$$u_0 = \frac{P_2}{A_0 E}\left[\frac{1}{m_1^2}(\sinh m_1 x - m_1 x \cosh m_1 L)(t_1 C_{21} + t_2 D_{21})\right.$$

$$\left. + \frac{1}{m_2^2}(\sinh m_2 x - m_2 x \cosh m_2 L)(t_1 C_{22} + t_2 D_{22})\right]$$

$$u_1 = \frac{P_2 t_1}{A_1 E}\left[\frac{C_{21}}{m_1^2}(m_1 x \cosh m_1 L - \sinh m_1 x) + \frac{C_{22}}{m_2^2}(m_2 x \cosh m_2 L - \sinh m_2 x)\right]$$

$$u_2 = \frac{P_2 x}{A_2 E} + \frac{P_2 t_2}{A_2 E}\left[\frac{D_{21}}{m_1^2}(m_1 x \cosh m_1 L - \sinh m_1 x) + \frac{D_{22}}{m_2^2}(m_2 x \cosh m_2 L - \sinh m_2 x)\right]$$

A combination to these solutions through superposition can be obtained wherein joints (1), (2), (3), (4), and (5) remain fixed as joint (0) is loaded. This combined solution gives the following results:

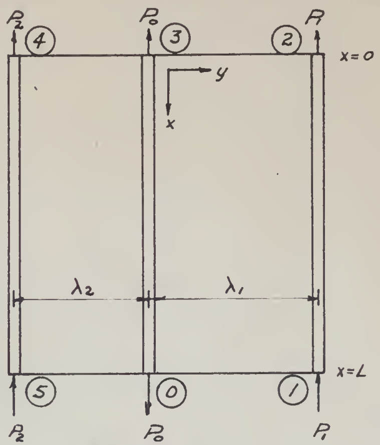


Figure 4

$$P_{o(x=0)} = P_o\left[1 - t_1(\gamma_1\delta_1 + \gamma_2\delta_2) - t_2(\gamma_1\delta_2 + \gamma_2\delta_4)\right]$$

$$P_{1(x=0)} = P_o\left[t_1(\gamma_1\delta_1 + \gamma_2\delta_2) - \beta_1'\right]$$

$$P_{2(x=0)} = P_o\left[t_2(\gamma_1\delta_2 + \gamma_2\delta_4 - \beta_2'\right]$$

$$P_{1(x=L)} = P_o\beta_1'$$

$$P_{2(x=L)} = P_o\beta_2'$$

The constants used in the foregoing developments are defined as follows:

$$K_1^2 = \frac{G t_1}{\lambda_1 E}\left[\frac{A_o + A_1}{A_o A_1}\right]$$

$$K_2^2 = \frac{G t_2}{\lambda_2 E}\left[\frac{A_o + A_2}{A_o A_2}\right]$$

$$\beta^2 = \frac{A_o A_2}{(A_o + A_2)(A_o + A_1)} - 1$$

$$m_1^2 = \frac{K_1^2 + K_2^2}{2} + \sqrt{\frac{1}{4}(K_1^2 + K_2^2)^2 - \beta^2 K_1^2 K_2^2}$$

$$m_2^2 = \frac{K_1^2 + K_2^2}{2} - \sqrt{\frac{1}{4}(K_1^2 + K_2^2)^2 - \beta^2 K_1^2 K_2^2}$$

$$Z_1^2 = \frac{G t_1}{\lambda_1 E}\left(\frac{A_o + A_1 + A_2}{A_o A_1}\right)$$

$$Z_2^2 = \frac{G t_2}{\lambda_2 E}\left(\frac{A_o + A_1 + A_2}{A_o A_2}\right)$$

$$C_{o1} = \frac{G\cdot}{A_o\lambda_1 E}\left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2}\right]\left[1 - \frac{m_2^2}{Z_1^2}\right]$$

$$C_{02} = \frac{G}{A_0 \lambda_1 E} \left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2} \right] \left[\frac{m_1^2}{z_1^2} - 1 \right]$$

$$D_{01} = \frac{G}{A_0 \lambda_2 E} \left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2} \right] \left[1 - \frac{m_2^2}{z_2^2} \right]$$

$$D_{02} = \frac{G}{A_0 \lambda_2 E} \left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2} \right] \left[\frac{m_1^2}{z_2^2} - 1 \right]$$

$$C_{11} = \frac{G}{A_1 \lambda_1 E} \left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2} \right] \left[\frac{A_0 + A_2}{A_0} \frac{m_2^2}{z_1^2} - 1 \right]$$

$$C_{12} = \frac{G}{A_1 \lambda_1 E} \left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2} \right] \left[1 - \frac{A_0 + A_2}{A_0} \frac{m_1^2}{z_1^2} \right]$$

$$D_{11} = \frac{-G}{A_0 \lambda_2 E} \left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2} \right] \left[\frac{m_2^2}{z_2^2} \right]$$

$$D_{12} = \frac{G}{A_0 \lambda_2 E} \left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2} \right] \left[\frac{m_1^2}{z_2^2} \right]$$

$$C_{21} = \frac{-G}{A_0 \lambda_1 E} \left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2} \right] \left[\frac{m_2^2}{z_1^2} \right]$$

$$C_{22} = \frac{G}{A_0 \lambda_1 E} \left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2} \right] \left[\frac{m_1^2}{z_1^2} \right]$$

$$D_{21} = \frac{G}{A_2 \lambda_2 E} \left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2} \right] \left[\frac{A_0 + A_1}{A_0} \frac{m_2^2}{z_2^2} - 1 \right]$$

$$D_{22} = \frac{G}{A_2 \lambda_2 E} \left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2} \right] \left[1 - \frac{A_0 + A_1}{A_0} \frac{m_1^2}{z_2^2} \right]$$

$$\alpha_1 = \frac{1}{m_1^2} \left(m_1 L \cosh m_1 L - \sinh m_1 L \right)$$

$$\alpha_2 = \frac{1}{m_2^2} \left(m_2 L \cosh m_2 L - \sinh m_2 L \right)$$

$$\beta_1' = \frac{(C_{01}\alpha_1 + C_{02}\alpha_2)(L/t_2 + D_{21}\alpha_1 + D_{22}\alpha_2) - (D_{01}\alpha_1 + D_{02}\alpha_2)(C_{21}\alpha_1 + C_{22}\alpha_2)}{(L/t_1 + C_{11}\alpha_1 + C_{12}\alpha_2)(L/t_2 + D_{21}\alpha_1 + D_{22}\alpha_2) - (D_{11}\alpha_1 + D_{12}\alpha_2)(C_{21}\alpha_1 + C_{22}\alpha_2)}$$

$$\beta_2' = \frac{(D_{01}\alpha_1 + D_{02}\alpha_2)(L/t_1 + C_{11}\alpha_1 + C_{12}\alpha_2) - (D_{11}\alpha_1 + D_{12}\alpha_2)(C_{01}\alpha_1 + C_{02}\alpha_2)}{(L/t_1 + C_{11}\alpha_1 + C_{12}\alpha_2)(L/t_2 + D_{21}\alpha_1 + D_{22}\alpha_2) - (D_{11}\alpha_1 + D_{12}\alpha_2)(C_{21}\alpha_1 + C_{22}\alpha_2)}$$

$$\delta_1 = C_{01} - \beta_1' C_{11} - \beta_2' C_{21}$$

$$\delta_2 = D_{01} - \beta_1' D_{11} - \beta_2' D_{21}$$

$$\delta_3 = C_{02} - \beta_1' C_{12} - \beta_2' C_{22}$$

$$\delta_4 = D_{02} - \beta_1' D_{12} - \beta_2' D_{22}$$

$$\gamma_1 = \frac{1}{m_1}(\cosh m_1 L - 1)$$

$$\gamma_2 = \frac{1}{m_2}(\cosh m_2 L - 1)$$

A solution for a two-stringer panel problem is also necessary for use in the load distribution method of solution of the more complicated panels. This solution was first developed by Mr. Paul Kuhn in reference (a). It can be shown that Professor Schneyer's solution for a three-stringer panel will reduce identically to Kuhn's two-stringer solution for the special case that either t_1 or t_2 equals zero. Kuhn's solution is given below.

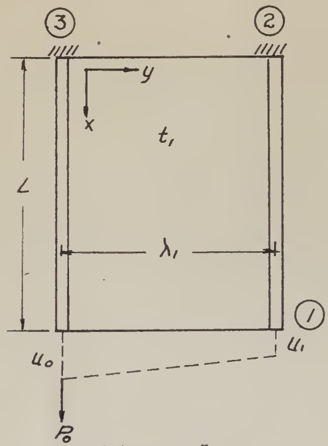


Figure 5

$$\tau_1 = \frac{P_0}{A_0} \frac{G}{E\lambda_1 K_1} \left(\frac{\sinh k_1 x}{\cosh k_1 x} \right)$$

$$\sigma_0 = \frac{P_0}{A_0} - \frac{A_1}{A_0} \frac{P_0}{A_0 + A_1} \left(1 - \frac{\cosh k_1 x}{\cosh k_1 L} \right)$$

$$\sigma_1 = \frac{P_0}{A_0 + A_1} \left[1 - \frac{\cosh k_1 x}{\cosh k_1 L} \right]$$

$$u_0 = \frac{P_0 X}{A_0 E} - \frac{A_1}{A_0} \frac{P_0}{(A_0+A_1)E} \left(X - \frac{\sinh k_1 X}{k_1 \cosh k_1 L} \right)$$

$$u_1 = \frac{P_0}{(A_0 + A_1)E} \left(X - \frac{\sinh k_1 X}{k_1 \cosh k_1 L} \right)$$

For the condition that joints (1), (2), and (3) remain fixed during loading we have:

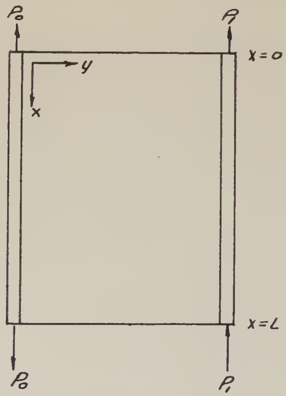


Figure 6

$$P_{0(x=0)} = P_0 \left[\frac{1 + \frac{A_0}{A_1} \frac{\sinh k_1 L}{k_1 L}}{\cosh k_1 L + \frac{A_0}{A_1} \frac{\sinh k_1 L}{k_1 L}} \right]$$

$$P_{1(x=0)} = P_0 \left[\frac{\frac{\sinh k_1 L}{k_1 L} - 1}{\cosh k_1 L + \frac{A_0}{A_1} \frac{\sinh k_1 L}{k_1 L}} \right]$$

$$P_{1(x=L)} = P_0 \left[\frac{1 - \dfrac{\tanh k_1 L}{k_1 L}}{1 + \dfrac{A_0}{A_1} \dfrac{\tanh k_1 L}{k_1 L}} \right]$$

K_1 in this solution is defined as in the three-stringer case, i.e.,

$$k_1^2 = \frac{G t_1}{\lambda_1 E} \left[\frac{A_0 + A_1}{A_0 A_1} \right]$$

EQUIPMENT AND PROCEDURE

Experimental tests were performed in the laboratory on a three-stringer and a four-stringer model with no cutouts, and on a four-stringer model with a cutout. Physical dimensions of these three specimens are given in Figs. (10), (11), and (12) of this report. All specimens were constructed of 24-ST aluminum alloy.

The specimens were mounted on a rigid frame structure, shown in Fig. (19). This structure was so designed that its deflections were of second order in comparison with the deflections of the test specimens under loading conditions. Attachment of the specimens to the frame was accomplished by means of 5/16 inch AN bolts running through the upper ends of the stringers. These bolts were tightened as securely as possible to insure the greatest possible degree of fixity at the attachment points.

Loading of the specimens was accomplished by means of a turnbuckle system employing the use of a tension dynamometer. As indicated in Fig. (~~20~~ 21), the loading on the three-stringer specimen was direct to the center stringer, whereas in the case of the four-stringer specimen, simultaneous loading at four points was obtained through the use of the special rig shown in Fig. (~~21~~ 20).

This rig was designed to give a loading ratio of two to one (2:1) between the inboard and outboard stringers.

Strain gage type tension and compression dynamometers were calibrated in a tensile testing machine and were found to have linear characteristics of load vs. strain throughout the desired loading range.

SR-4 strain gages were placed on the model at desired locations. Types of strain gages used were A-1, A-7, and A-10. Space limitations dictated the size of gage to be used at a given location. To eliminate the effects of bending, a duplicate set of gages was located on the back face of all specimens. Two Young strain indicators were used to obtain strain readings. A system of circular type electrical sweep switches was used in conjunction with the strain recorders.

The experimental procedure in itself consisted essentially of straining the tension dynamometer to a desired load, allowing the system to come to equilibrium at that load, and recording the strain readings at the various stations throughout the model.

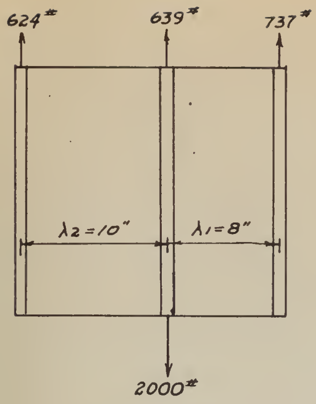
RESULTS AND DISCUSSION

Since tests were performed on three distinct specimens, it is believed that a separate discussion of each would be the most effective manner of presentation of results.

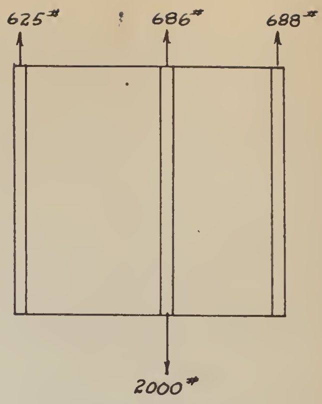
Case I. Three-stringer specimen.

For this case the problem was divided into two phases: (a) Loading on center stringer with no restraint on the lower outboard stringers, and (b) loading on center stringer with lower outboard stringers restrained from movement.

Data was taken for phase (a) with loadings of both 1000 and 2000 pounds. This data appears in Table I. It should be noted that linearity was exact within the limit of accuracy of measurement. Reduction of this data for the 2000 pound loading condition gave the results indicated below. Theoretical calculations for this model were made, and these results appear alongside the experimental results. Effective width of sheet was not considered in these theoretical calculations since only the internal load points were being considered, and it was believed that in a simple panel of this type the external loads would not be vitally affected by the exclusion of the effective width considerations.



Theoretical



Experimental

Figure 7

The percentage discrepancy between theoretical and experimental, based on theoretical values, are

P_o	6.8%
P_1	7.1%
P_2	0.1%

These results appear to give good agreement with theory.

In case (b), the center stringer was loaded to 2000 pounds, and the outboard lower stringer points were restrained. Load readings for these outer points were obtained by use of compression dynamometers. Inasmuch as the elongation at these outer points was of an order of magnitude of less than .002 inches, coupled with the fact that the dynamometer support system was not entirely

rigid to this degree of fineness, part of the load which should have been read on the dynamometers was picked up at the upper supports. As an alternative solution, it was then decided to take the loads actually read by the compression dynamometers, and to calculate the values of load at the upper stringer points due to each of these lower loads acting independently. If the theory was correct, these solutions, when added to the solution of the 2000 pound load of case (a), should then match the actual load readings experimentally determined at the upper stringer points. Data for this run is given in Table II. The following results ensued from this procedure.

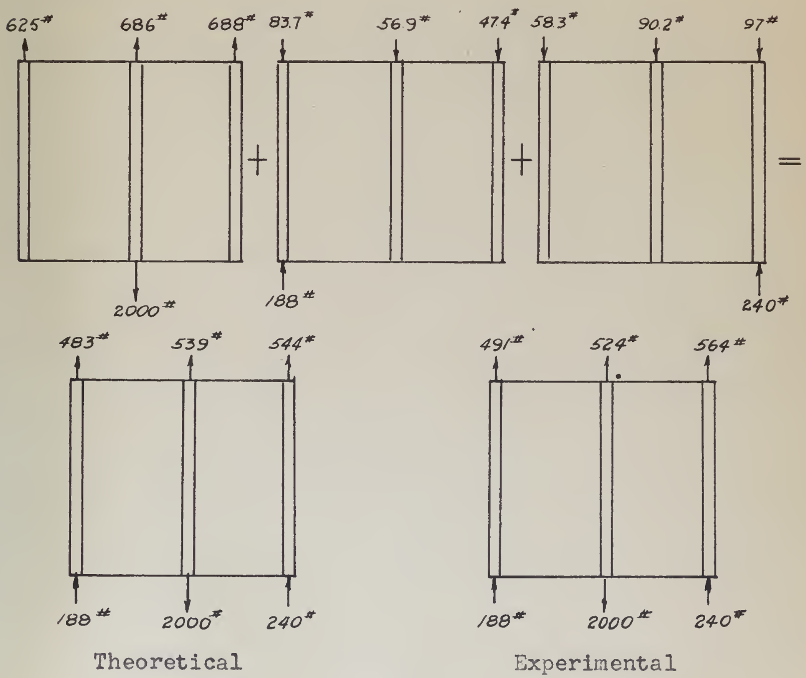


Figure 8

The percentage discrepancy between theoretical and experimental values for each load are:

P_o	2.8%
P_1	3.7%
P_2	1.6%

From these results, it is evident that the original assumption of superposition of loading has been proven valid.

Two shear gages were mounted at the third points of the 8" panel. During the test it was discovered that the lower shear gage was inoperative, and therefore, only the shear value at the upper third point could be determined. The experimental value of this shear was determined to be 512 psi. The theoretical value computed was 127 psi. Since only one point was involved, it is difficult to draw any conclusions from the apparent wide discrepancy in these results.

Case II.

For this portion of the investigation, a symmetrical four-stringer model with no cutouts was used. A load of 2000 pounds was applied through the tension dynamometer. Initially, strain gages were placed at the third points of all four stringers. One gage in each set was secured to the stringer proper, while its mate on the opposite side was secured to the sheet. As before, gages were also located on both sides of the upper stringers, each gage being secured to the stringer itself. To determine

the effective width of sheet to be added to the stringer areas, fifteen strain gages were located spanwise on the sheet across the upper third point station as indicated in Fig. (23). Six additional spanwise gages were placed along the lower third point station.

The experimental data taken for this model appears in Table III. From the upper third point strain readings, curves showing strain distribution across the sheet were plotted and appear in Fig. (13). The integrated area under this curve, when multiplied by the factor (Et), i.e. modulus times sheet thickness, gave the total load carried by the sheet. The load is indicated on the above mentioned figure. The load carried by half the sheet width in each panel was assigned to its adjacent stringer at the experimentally determined stringer stress, thus forming the basis for obtaining effective width of sheet. Fifty-percent of the four-inch sheet adjacent to the outboard stringers was determined to be effective while only forty-percent of the nine-inch sheet adjacent to both sides of the inboard stringers was effective. In the theoretical calculations these effective widths were assumed constant throughout the entire length of the stringer.

Computation of stringer loads at the third point stations, based on stringer strain gage readings, when added to the load carried by the sheet, failed to equal the applied loadings of 2000 pounds by a wide margin.

It was concluded that the combination of stringer gages, one on the stringer and one on the sheet, gave erroneous readings of the stringer loads, and therefore, an alternate method of determination of stringer loads was necessary. The smooth plot of sheet strain distribution, Fig. (13), was projected to the rivet line of each stringer. This point then represented a stringer gage reading on the sheet face. From the difference in the gage readings of each set of gages mounted on opposite sides of the sheet, the bending moment normal to the sheet was determined. A plot of this cross-panel moment distribution is given in Fig. (14). In the same manner as above, these plots were projected to the rivet lines of each stringer, and these projected points then represented the bending moment at the stringer third points. Since the stringer thickness was three times that of the sheet, the strain on the free face of the stringer, due to bending, is three times that on the sheet face of the stringer. By using this relation, a value of strain reading on the free face of the stringer could be obtained. By averaging the strain readings thus obtained, bending effects were eliminated, and the load in the stringer determined. The stringer loads determined by this method, when added to the load carried by the sheet, totaled 1990 pounds compared to the applied load of 2000 pounds, an error of only 1/2%. A similar method of

obtaining stringer loads was applied to the lower third point stations. Since fewer strain gages were employed, an assumption was made that these curves would be of a similar type to those of the upper third points.

Theoretical computations of this problem were made both with and without effective sheet width being considered. In order to compare theoretical and experimental results at the interior stringer stations, it is necessary to compare stresses rather than loads. Results of experimental and theoretical calculations for a 2000 pound loading are given in Fig. (15). In this figure, the theoretical calculations are presented in graphical form as stress distribution along each of the four stringers. Two theoretical curves are plotted for each stringer, one representing the consideration of effective width and the other neglecting effective width. Experimentally determined stresses are plotted at the respective points under consideration.

From a study of the Fig. (15), it is immediately apparent that the effective width varies along the stringer. Since the experimentally determined stresses at the stringer ends coincide almost exactly with those stresses calculated without consideration of effective width, it may be concluded that the effective width is zero at the ends of the panel. At the upper third point stations, the experimental results are in very good

agreement with those calculated through the inclusion of effective width and are obviously far from the results predicted when effective width is neglected.

The experimental results at the lower third point stations also indicate that there is a similar effective width which must be considered. Although discrepancies between theoretical and experimental values exist here on three of the four stringers, it is noted that the experimental points lie much closer to the results in which effective width was considered. It appears reasonable to assume that the effective widths at the lower third points should be identical with those of the upper third points due to symmetry existing in the model. The discrepancies existing between the experimental and theoretical results are believed to lie in the experimental results and may be attributed to the following reasons:

(1) The lack of symmetry in strain readings taken at these points, as evidenced by their plot on Fig. (16).

(2) The fact that an insufficient number of strain readings were taken across the panel at this station to give an accurate description of the strain conditions existing in the sheet. The stringer strains computed from these few readings may be viewed with some suspicion.

Case III.

For this case, the outboard center third panel on one side of the model used in Case II was cut out, as shown in Fig. (24). In computation of the theoretical

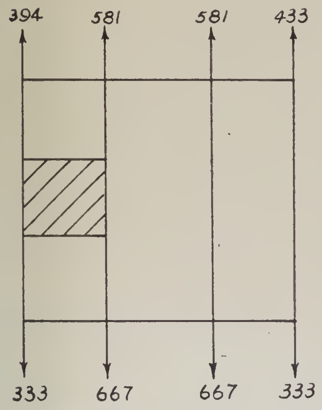
problem for this case, the same effective widths utilized in Case II were used, excepting in the area of the cut-out where proper modification was made. Effective width was assumed acting throughout the length of the stringers. No independent calculations were made neglecting effective width for this case. Complete theoretical calculations for this case are presented in the section of this report entitled "Sample Calculations." The methods involved in this more complicated case completely cover methods used in computations for Cases I and II.

All strain gages used in Case II with exception of those removed by the cutout were again utilized. An additional set of two strain gages was placed at the center of the outboard stringer adjacent to the cutout. A small piece of the sheet remaining on the stringer was removed to accomodate placing of the strain gage directly against the stringer face, thus avoiding the complications discussed in Case II in determining stringer strains. Experimental data for this model is given in Table IV.

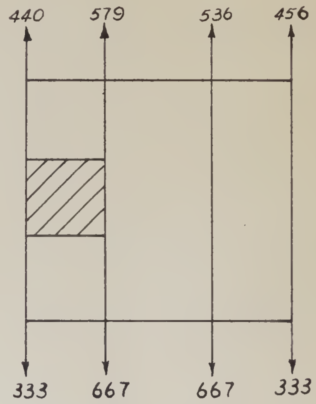
Strain and moment distribution curves for this case are shown in Figures (16) and (17). As in Case II, the experimental and theoretical results for this model are presented in graphical form in Fig. (18). The theoretical plot of stress on stringers (1) and (2) show a discontinuity at the third points. This is due to the fact

that effective width is assumed present on the sheet side, and is, of course, zero at the cutout.

At the exterior stringer points it is noted that the experimental results of stress do not agree with the theoretical results based on effective width consideration. If theoretical calculations had been made with no effective width at these points, the experimental stresses would have been in closer agreement with the theoretical stress. This can be shown by considering loads rather than stresses at these points as indicated in the figure below, keeping in mind that the theoretical loads represent consideration of effective width.



Theoretical



Experimental

Figure 9

At the third point stations, the experimental results again indicate that the effect of effective width of sheet is present in the interior of the panel. On stringer (1), it is noted that the experimental stress value coincides exactly with the theoretical value. It was on this stringer that the strain gages were placed directly against the stringer faces, thus providing a much higher degree of accuracy of strain measurements. Discrepancies existing at the other stringer points are believed to lie mainly in the actual strain measurements taken since total computed load carried by the stringers and the sheet, based on these measurements, failed to account for 10% of the known applied load. New attempts were made to account for this discrepancy in total load with no success. Due to shortage of time, a full investigation of this discrepancy was not possible.

No results of shear investigation in the sheet are presented for this case, since one of the two sets of gages again proved to be defective. Time limitations prevented a complete investigation of the shear phenomena within the sheet.

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions are drawn form this investigation:

(1) Commensurate with the degree of accuracy of experimental measurements in this investigation, it is firmly believed that the theory developed by Professor Schneyer gives an accurate analysis of the axial stresses existing in panels with and without cutouts.

(2) In view of the incomplete investigation of shear existing in the sheet, no direct conclusions can be made regarding the actual state of shear within the sheet. However, since the stringer loads are a direct function of the shear transmission in the sheet, and since the predicted stringer loads have been proven valid, it appears reasonable to assume that the predicted state of shear in the sheet is correct.

(3) The principle of superposition, a basic assumption of this theory, was proven to be entirely valid within the elastic limit.

(4) For dependable measurements of stringer strains, the strain gages should be placed directly against the stringer faces. In further investigation of this shear lag problem, it is recommended that a small section of the sheet riveted to the stringer be removed to accomo-

date a strain gage. It is believed that any local effects arising from this small cutout will be minor.

(5) Lack of symmetry in strain readings in a geometrically similar model may be attributed in part to local variations in the manner in which the rivets transfer loads from the sheet to the stringers. In addition, in spite of extreme care taken in the construction of the symmetrical model, it is probable that some small deviations were present.

(6) For the particular models used in this investigation, it appears that only fifty-percent of the sheet adjacent to an exterior stringer and only forty-percent adjacent to an interior stringer is effective at interior stations. In the solution of any problem involving shear lag, it is deemed essential that some estimate be made regarding the variation of effective width of sheet to be added to the stringer areas. The effective width has been proven conclusively to be equal to zero at the ends of the panels. However, it is believed that it approaches its maximum value within a short distance of the ends. To determine the actual variation of effective width, it would be necessary to run a series of strain measurements across the sheet at a number of different longitudinal stations. For the case of compression loading, consideration of effective width is materially altered from that of tension loading due to buckling effects.

In final conclusion, it is desired to state the sincere belief of the writers that Professor Schneyer's work on the shear lag problem represents a highly substantial contribution to this field.

REFERENCE

(a) Kuhn, Paul. "Stress Analysis of Beams With Shear Deformation of the Flanges," NACA Technical Report No. 608.

SAMPLE CALCULATIONS

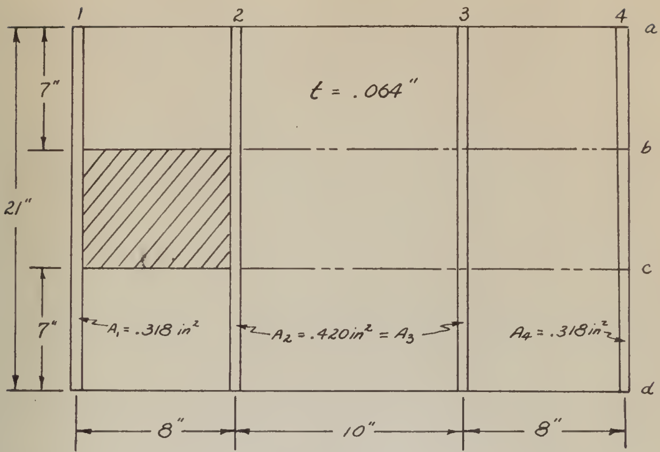
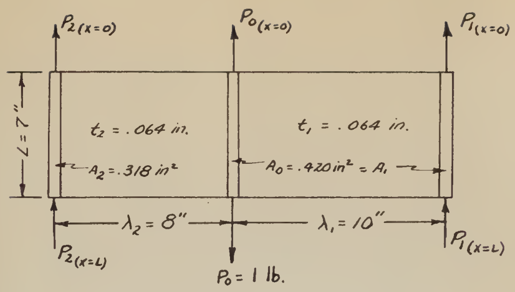


Figure (a)

The basic dimensions of the four-stringer panel with a cut-out are shown in Figure (a). Since loads in the stringers are desired at the third points, the panel is divided into three 7" lengths as indicated. To apply the load distribution method of solution to this panel it is necessary to compute the load distribution factors at each point. To accomplish this, the panel must be treated as a series of basic panels for which solutions have been achieved.

Considering first the three-stringer panel 1c-3c-1d-3d, which is identical to panel 1a-3a-1b-3b, the Schneyer solution is applied as follows:



$$K_1^2 = \frac{G t_1}{\lambda_1 E}\left[\frac{A_0 + A_1}{A_0 A_1}\right] = \frac{.381 \times .064}{10}\left[\frac{.420 + .420}{(.420)^2}\right] = .0116$$

$$K_1 = .1077$$

$$K_2^2 = \frac{G t_2}{\lambda_2 E}\left[\frac{A_0 + A_2}{A_0 A_2}\right] = \frac{.381 \times .064}{8}\left[\frac{.420 + .318}{(.420)(.318)}\right] = .01684$$

$$K_2 = .1299$$

$$\beta^2 = \frac{A_0 A_2}{(A_0 + A_2)(A_0 + A_1)} - 1 = \frac{.420 \times .318}{(.738)(.840)} - 1 = -0.7846$$

$$m_1^2 = \frac{K_1^2 + K_2^2}{2} + \sqrt{\frac{1}{4}(K_1^2 + K_2^2) + \beta^2 K_1^2 K_2^2} = \frac{.02844}{2} + \sqrt{\frac{(.02844)^2}{4} - .7486 \times .0116 \times .01684}$$

$$= .02124$$

$$m_1 = .1459$$

$$m_2^2 = \frac{K_1^2 + K_2^2}{2} - \sqrt{\frac{1}{4}(K_1^2 + K_2^2) + \beta^2 K_1^2 K_2^2} = .0072$$

$$m_2 = .0848$$

$$Z_1^2 = \frac{G t_1}{\lambda_1 E}\left(\frac{A_1 + A_2 + A_3}{A_0 A_1}\right) = \frac{.381 \times .064}{10}\left(\frac{1.158}{(.42)^2}\right) = .0160$$

$$Z_2^2 = \frac{G t_2}{\lambda_1 E}\left(\frac{A_1 + A_2 + A_3}{A_0 A_1}\right) = \frac{.381 \times .064}{8}\left(\frac{1.158}{(.42)^2}\right) = .020$$

$m_1 L = .1459 \times 7 = 1.0213$ $\qquad$ $m_2 L = .0848 \times 7 = .5936$

$$\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2} = \frac{.1459 \times .6375}{.01404} = 6.62$$

$$\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2} = \frac{.0848 \times .846}{.01404} = 5.11$$

$$\frac{m_2^2}{z_1^2} = \frac{.0072}{.016} = .45 \qquad \frac{G}{A_0 \lambda_1 E} = \frac{.381}{.420 \times 10} = .0907$$

$$\frac{m_2^2}{z_2^2} = \frac{.0072}{.020} = .36 \qquad \frac{G}{A_2 \lambda_2 E} = \frac{.381}{.318 \times 8} = .1499$$

$$\frac{m_1^2}{z_1^2} = \frac{.02124}{.016} = 1.33 \qquad \frac{G}{A_1 \lambda_1 E} = \frac{.381}{.420 \times 10} = .0907$$

$$\frac{m_1^2}{z_2^2} = \frac{.02124}{.02} = 1.062 \qquad \frac{G}{A_0 \lambda_2 E} = \frac{.381}{.420 \times 8} = .1134$$

$$C_{01} = \frac{G}{A_0 \lambda_1 E}\left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2}\right]\left[1 - \frac{m_2^2}{z_1^2}\right] = .0907 \times 6.62 \times .55 = .330$$

$$C_{02} = \frac{G}{A_0 \lambda_1 E}\left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2}\right]\left[\frac{m_1^2}{z_1^2} - 1\right] = .0907 \times 5.11 \times .33 = .1532$$

$$D_{01} = \frac{G}{A_0 \lambda_2 E}\left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2}\right]\left[1 - \frac{m_2^2}{z_2^2}\right] = .1134 \times 6.62 \times .64 = .480$$

$$D_{02} = \frac{G}{A_0 \lambda_2 E}\left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2}\right]\left[\frac{m_1^2}{z_2^2} - 1\right] = .1134 \times 5.11 \times .062 = .0360$$

$$C_{11} = \frac{G}{A_1 \lambda_1 E}\left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2}\right]\left[\frac{A_0 + A_2}{A_0} \frac{m_2^2}{z_1^2} - 1\right] = .0907 \times 6.62 \times (-.209) = -.1255$$

$$C_{12} = \frac{G}{A_1 \lambda_1 E}\left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2}\right]\left[1 - \frac{A_0 + A_2}{A_0} \frac{m_1^2}{z_1^2}\right] = .0907 \times 5.11 (-1.34) = -.622$$

$$D_{11} = -\frac{G}{A_0 \lambda_2 E}\left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2}\right]\left[\frac{m_2^2}{z_2^2}\right] = -.1134 \times 6.62 \times .36 = -.270$$

$$D_{12} = \frac{G}{A_0 \lambda_2 E}\left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2}\right]\left[\frac{m_1^2}{z_2^2}\right] = .1134 \times 5.11 \times 1.062 = .616$$

$$C_{21} = \frac{-G}{A_0 \lambda_1 E}\left[\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2}\right]\left[\frac{m_2^2}{z_1^2}\right] = -.0907 \times 6.62 \times .45 = -.270$$

$$C_{22} = \frac{G}{A_0 \lambda_1 E}\left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2}\right]\left[\frac{m_1^2}{z_1^2}\right] = .0907 \times 5.11 \times 1.33 = .617$$

$$D_{21} = \frac{G}{A_2 \lambda_2 E}\left\{\frac{m_1 \operatorname{sech} m_1 L}{m_1^2 - m_2^2}\right]\left[\frac{A_0 + A_1}{A_0}\frac{m_2^2}{z_2^2} - 1\right] = .1499 \times 6.62(-.28) = -.2775$$

$$D_{22} = \frac{G}{A_2 \lambda_2 E}\left[\frac{m_2 \operatorname{sech} m_2 L}{m_1^2 - m_2^2}\right]\left[1 - \frac{A_0 + A_1}{A_0}\frac{m_1^2}{z_2^2}\right] = .1499 \times 5.11(-1.124) = -.861$$

$$\alpha_1 = \frac{1}{m_1^2}\left(m_1 L \cosh m_1 L - \sinh m_1 L\right) = \frac{1}{.02124}(1.0213 \times 1.5685 - 1.2083)$$

$$= 18.5$$

$$\alpha_2 = \frac{1}{m_2^2}\left(m_2 L \cosh m_2 L - \sinh m_2 L\right) = \frac{1}{.0072}(.5936 \times 1.1815 - .6291)$$

$$= 10.02$$

$$C_{01}\alpha_1 + C_{02}\alpha_2 = .330 \times 18.5 + .1532 \times 10.02 = 7.64$$

$$\frac{L}{t_2} + D_{21}\alpha_1 + D_{22}\alpha_2 = 109.4 - .2775 \times 18.5 - .861 \times 10.02 = 95.64$$

$$D_{01}\alpha_1 + D_{02}\alpha_2 = .480 \times 18.5 + .036 \times 10.02 = 9.24$$

$$C_{21}\alpha_1 + C_{22}\alpha_2 = -.270 \times 18.5 + .617 \times 10.02 = 1.19$$

$$\frac{L}{t_1} + C_{11}\alpha_1 + C_{12}\alpha_2 = 109.4 - .1255 \times 18.5 - .622 \times 10.02 = 100.9$$

$$D_{11}\alpha_1 + D_{12}\alpha_2 = -.270 \times 18.5 + .616 \times 10.02 = 1.17$$

$$\beta_1' = \frac{(C_{01}\alpha_1 + C_{02}\alpha_2)\left(\frac{L}{t_2} + D_{21}\alpha_1 + D_{22}\alpha_2\right) - (D_{01}\alpha_1 + D_{02}\alpha_2)(C_{21}\alpha_1 + C_{22}\alpha_2)}{(L/t_1 + C_{11}\alpha_1 + C_{12}\alpha_2)(L/t_2 + D_{21}\alpha_1 + D_{22}\alpha_2) - (D_{11}\alpha_1 + D_{12}\alpha_2)(C_{21}\alpha_1 + C_{22}\alpha_2)}$$

$$= \frac{7.64 \times 95.64 - 9.24 \times 1.19}{95.64 \times 100.9 - 1.17 \times 1.19} = .0745$$

$$\beta_2' = \frac{(D_{01}\alpha_1 + D_{02}\alpha_2)(L/t_1 + C_{11}\alpha_1 + C_{12}\alpha_2) - (D_{11}\alpha_1 + D_{12}\alpha_2)(C_{01}\alpha_1 + C_{02}\alpha_2)}{(L/t_1 + C_{11}\alpha_1 + C_{12}\alpha_2)(L/t_2 + D_{21}\alpha_1 + D_{22}\alpha_2) - (D_{11}\alpha_1 + D_{12}\alpha_2)(C_{21}\alpha_1 + C_{22}\alpha_2)}$$

$$= \frac{9.24 \times 100.9 - 1.17 \times 7.64}{95.64 \times 100.9 - 1.17 \times 1.19} = .0955$$

$$\delta_1 = C_{01} - \beta_1' C_{11} - \beta_2' C_{21} = .330 + .0745 \times .1255 + .0956 \times .270 = .3652$$

$$\delta_2 = D_{01} - \beta_1' D_{11} - \beta_2' D_{21} = .480 + .0745 \times .270 + .0956 \times .2775 = .5266$$

$$\delta_3 = C_{02} - \beta_1' C_{12} - \beta_2' C_{22} = .1532 + .0745 \times .622 - .0956 \times .617 = .1406$$

$$\delta_4 = D_{02} - \beta_1' D_{12} - \beta_2' D_{22} = .0360 - .0745 \times .616 + .0956 \times .861 = .0725$$

$$\gamma_1 = \frac{1}{m_1}(\cosh m_1 L - 1) = \frac{1}{.1459}(1.5685 - 1) = 3.90$$

$$\gamma_2 = \frac{1}{m_2}(\cosh m_2 L - 1) = \frac{1}{.0848}(1.1815 - 1) = 2.14$$

$$\gamma_1 \delta_1 + \gamma_2 \delta_3 = 3.9 \times .3652 + 2.14 \times .1406 = 1.725$$

$$\gamma_1 \delta_2 + \gamma_2 \delta_4 = 3.9 \times .5266 + 2.14 \times .0725 = 2.210$$

$$P_{0(x=0)} = P_0\left[1 - t_1(\gamma_1\delta_1 + \gamma_2\delta_3) - t_2(\gamma_1\delta_2 + \gamma_2\delta_4)\right]$$

$$= 1\left[1 - .064(1.725 + 2.210)\right] = .7483$$

$$P_{1(x=0)} = P_0\left[t_1(\gamma_1\delta_1+\gamma_2\delta_3)-\beta_1\right] = 1\left[.064\times 1.686-.0745\right]$$

$$= .0357$$

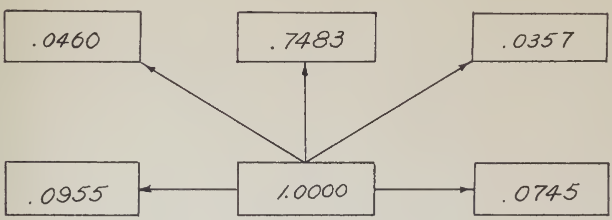
$$P_{2(x=0)} = P_0\left[t_2(\gamma_1\delta_2+\gamma_2\delta_4)-\beta_2\right] = 1\left[.064\times 2.210-.0955\right]$$

$$= .0460$$

$$P_{1(x=L)} = P_0\beta_1' = 1\times .0745 = .0745$$

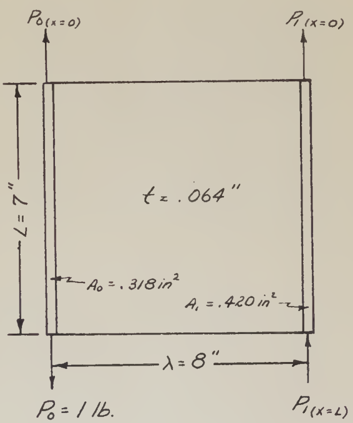
$$P_{2(x=L)} = P_0\beta_2' = 1\times .0955 = .0955$$

In the above solution all stringer end points are fixed except for the lower center stringer where the load P_0 is applied. By taking P_0 equal to one (1) lb., the remaining five "P" loads represent the distribution factors for the lower (and upper) center panel point. Its graphical representation is indicated below.



Since the three-stringer panels 2a-4a-2b-4b, 2b-4b-2c-4c and 2c-4c-2d-4d are mirror images of the panel solved above, the distribution factors computed will also hold for the lower and upper center panel points of these panels.

It is now necessary to find the distribution factors for the two-stringer panel 1c-2c-1d-2d. Here the points 1c, 2c and 2d remain fixed when a load at 1d is distributed. The distribution factors at 1c will be identical to that of 1d. Similarly, since the panels are identical, these distribution factors will also be valid at joints 1a, 1b, 4a, 4b, 4c and 4d. The Kuhn solution is applied to these panels as follows:



$$K^2 = \frac{Gt}{\lambda E}\left[\frac{A_0 + A_1}{A_0 A_1}\right] = \frac{.381 \times .064}{8}\left[\frac{.318 + .420}{.318 \times .420}\right] = .01684$$

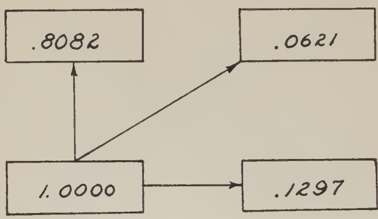
$$K = .1299$$

$$KL = .1299 \times 7 = .9093$$

$$P_{1(x=L)} = P_0 \left[\frac{1 - \dfrac{\tanh KL}{KL}}{1 + \dfrac{A_0}{A_1} \dfrac{\tanh KL}{KL}} \right] = 1 \left[\frac{1 - \dfrac{.7208}{.9093}}{1 + \dfrac{.318}{.420} \times \dfrac{.7208}{.9093}} \right] = .1297$$

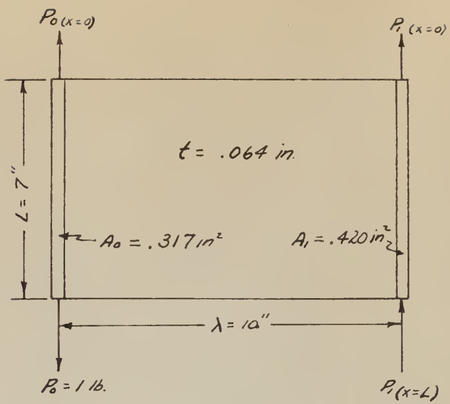
$$P_{0(x=0)} = P_0 \left[\frac{1 + \dfrac{A_0}{A_1} \dfrac{\sinh KL}{KL}}{\cosh KL + \dfrac{A_0}{A_1} \dfrac{\sinh KL}{KL}} \right] = 1 \left[\frac{1 + \dfrac{.318}{.420} \times \dfrac{1.0399}{.9093}}{1.4427 + \dfrac{.318}{.420} \dfrac{1.0399}{.9093}} \right] = .8082$$

$$P_{1(x=0)} = P_0 \left[\frac{\dfrac{\sinh KL}{KL} - 1}{\cosh KL + \dfrac{A_0}{A_1} \dfrac{\sinh KL}{KL}} \right] = 1 \left[\frac{\dfrac{1.0399}{.9093} - 1}{2.313} \right] = .0621$$



The distribution factors at joints 1b and 1c (cutout section) are equal to 1.00 since all the load at these joints must pass into the stringer, i.e., there is no distribution to joints 2b and 2c.

One panel remains for which distribution factors must be computed, 2b-3b-2c-3c. The Kuhn solution is applied to this panel to obtain the distribution factors for joints 2b and 2c. Its solution is indicated below:



$$K^2 = \frac{.381 \times .064}{10}\left[\frac{.317 + .420}{.317 \times .420}\right] = .0135$$

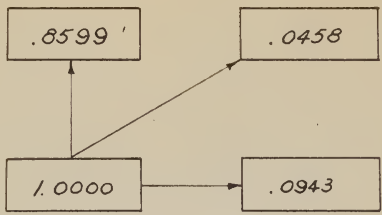
$$K = .116$$

$$K_1 L = .116 \times 7 = .812$$

$$P_{1\,(x=L)} = \frac{1 - \frac{.6375}{.812}}{1 + \frac{.317}{.420}\,\frac{.6375}{.812}} = .0943$$

$$P_{o\,(x=0)} = \frac{1 + \frac{.317}{.420} \times \frac{.8274}{.812}}{1.2979 + \frac{.317}{.420} \times \frac{.8274}{.812}} = .8599$$

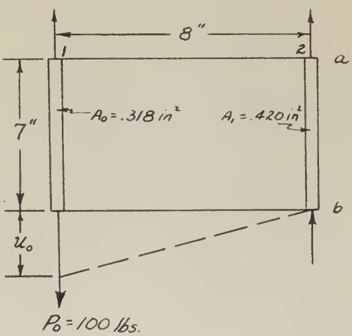
$$P_{1\,(x=0)} = \frac{\frac{.8274}{.812} - 1}{1.2979 + \frac{.317}{.420} \times \frac{.8274}{.812}} = .0458$$



All load distribution factors having been determined, it is now necessary to determine the distribution factors at the joints; i.e., as the loads accumulate in the upper and lower "boxes" of a joint, an unbalance of load is present. When the joint is unlocked, this unbalanced load must be distributed to the upper and lower "boxes" in such a manner that the joint is balanced. This is analogous to saying that when a section is cut at the joint the load on the lower free body must be equal and opposite to the load on the upper free body. To determine these joint distribution factors use is made of both the Schneyer and the Kuhn solutions for stringer displacement of three-stringer and two-stringer panels respectively. The upper free body is loaded with a known load, and the displacement is computed. Since the corresponding point of the lower free body must undergo an identical displacement, the load on the lower free body required to produce that displacement may be found. Then the ratio of the load in each free body to the sum of the two free body loads gives the desired joint distribution factor.

At joints 3b, 3c, 4b and 4c the joint distribution factors are .50 - .50 since the three-stringer panels from stringer (2) to stringer (4) are identical, and hence the load required to produce a given displacement at joints 3b and 3c are identical. The same argument applies to the two-stringer panels from stringer (3) to stringer (4).

The joint distribution factors at joints 1b and 2b are obtained as indicated below. The distribution factors at 1c and 2c are identical to those of 1b and 2b respectively.

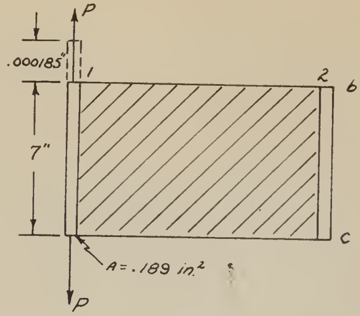


$$u_o = \frac{P_0 x}{A_0 E} - \frac{A_1}{A_0}\,\frac{P_0}{(A_0+A_1)E}\left(x - \frac{\sinh Kx}{K \cosh KL}\right)$$

@ $x = L$

$$u_o = \frac{100 \times 7}{.318 \times 10.5 \times 10^6} - \frac{.420}{.318}\,\frac{100}{.738 \times 10.5 \times 10^6}\left(7 - \frac{1.0399}{.1299 \times 1.4427}\right)$$

$$= .000185 \text{ inches}$$



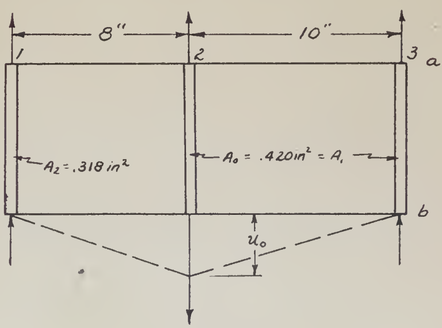
$$P = \frac{\delta AE}{L} = \frac{.000185 \times .189 \times 10.5 \times 10^6}{7} = 52.5 \text{ lbs.}$$

$$\text{UPPER DISTRIBUTION FACTOR} = \frac{100}{152.5} = .656$$

$$\text{LOWER DISTRIBUTION FACTOR} = \frac{52.5}{152.5} = .344$$

****

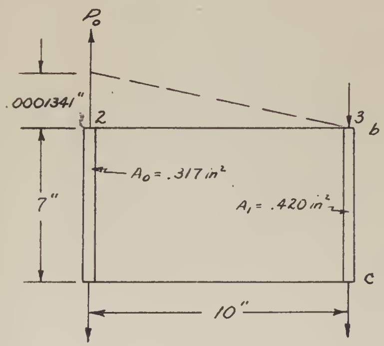
Considering now joints 2b and 2c, we have:



$$u_o = \frac{P_o}{A_o E}\left[x + \left\{\frac{1}{m_1^2}(\sinh m_1 x - m_1 x \cosh m_1 L)(t_1 C_{o1} + t_2 D_{o1}) + \frac{1}{m_2^2}(\sinh m_2 x - m_2 x \cosh m_2 L)(t_1 C_{o2} + t_2 D_{o2})\right\}\right]$$

$$= \frac{100}{.420 \times 10.5 \times 10^6}\left[7 + \left\{\frac{1}{.02124}(1.2003 - 1.0213 \times 1.5685)(.064)(.330 + .480) + \frac{1}{.0072}(.6291 - .5936 \times 1.1815)(.064)(.1532 + .0360)\right\}\right]$$

$$= .0001341 \text{ inches}$$



$$u_o = \frac{P_o x}{A_o E} - \frac{A_1}{A_o}\frac{P_o}{(A_o + A_1)E}\left(x - \frac{\sinh Kx}{K \cosh KL}\right)$$

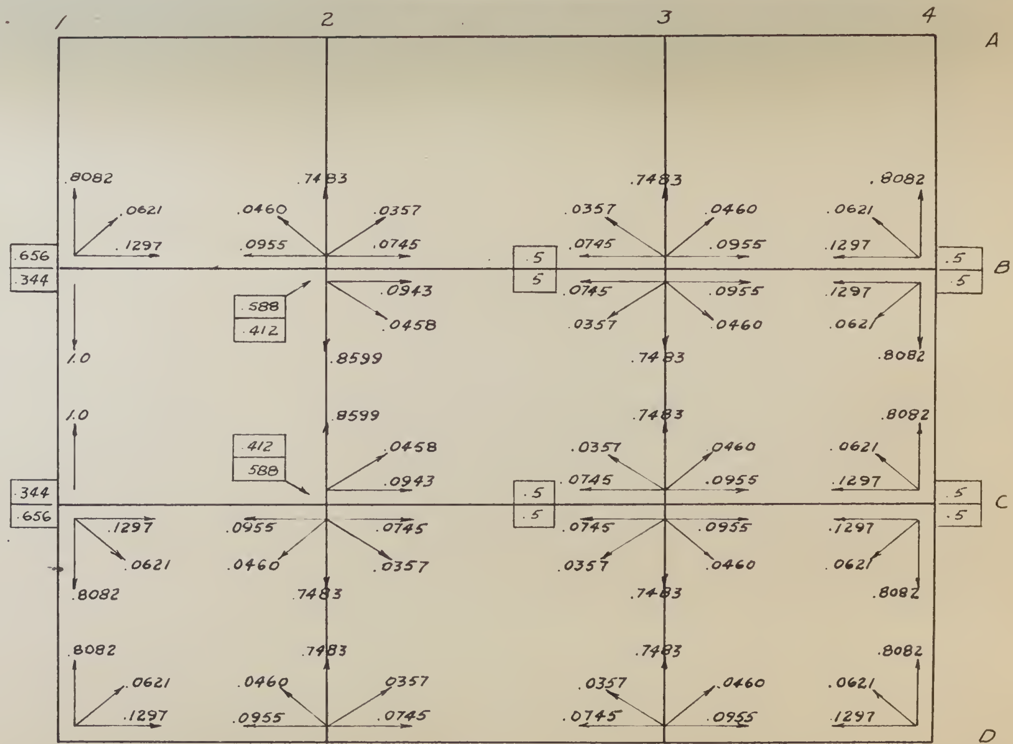
$$.0001341 = \frac{P_o \times 7}{.317 \times 10.5 \times 10^6} - \frac{.420}{.317}\frac{P_o}{.737 \times 10.5 \times 10^6}\left(7 - \frac{.8274}{.1077 \times 1.2979}\right)$$

$$P_o = 70 \text{ lbs.}$$

$$\text{UPPER DISTRIBUTION FACTOR} = \frac{100}{170} = .588$$

$$\text{LOWER DISTRIBUTION FACTOR} = \frac{70}{170} = .412$$

A complete picture of the load and joint distribution factors is given in Figure (b). By entering the given loadings of 333 lbs. at joints 1d and 4d and 667 lbs. at 2d and 3d, the loads are distributed in accordance with the factors of Figure (b) until all of the loads are run out to joints 1a, 2a, 3a and 4a. Final results of these calculations are given in Figure (c). All loadings shown in Figure (c) are in pounds.



1
2
3
4
A
B
C
D
.8082
.0621
.1297
.7483
.0460
.0955
.0357
.0745
.7483
.0357
.0745
.0460
.0955
.8082
.0621
.1297
.656
.344
.5
5
.5
.5
1.0
.588
.412
.0943
.0458
.8599
.0745
.0357
.0955
.0460
.7483
.1297
.0621
.8082
1.0
.412
588
.8599
.0458
.0943
.7483
.0357
.0745
.0460
.0955
.8082
.0621
.1297
.344
.656
.5
.5
.5
.5
.1297
.0621
.8082
.0955
.0460
.0745
.0357
.7483
.0745
.0357
.0955
.0460
.7483
.1297
.0621
.8082
.8082
.0621
.1297
.7483
.0460
.0955
0357
.0745
.7483
.0357
.0745
.0460
.0955
.8082
.0621
.1297

Figure (b)

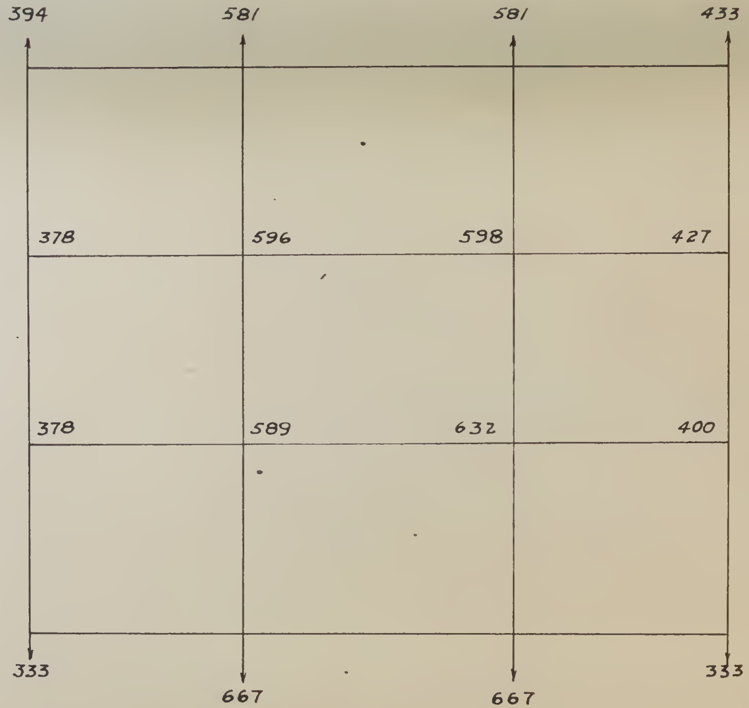
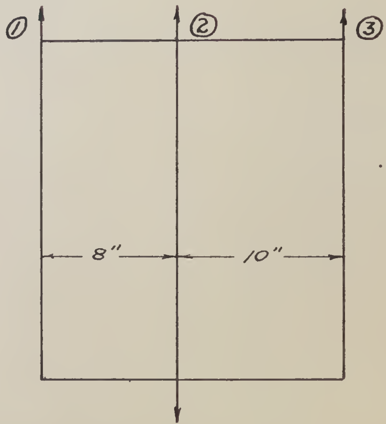


Figure (c)

LOAD	STRINGER (1)			STRINGER (2)			STRINGER (3)		
	1a	1b	1av	2a	2b	2av	3a	3b	3av
2000#	403	263	333	313	264	289	204	359	282
1000#	140	193	166	205	83	144	10	272	141

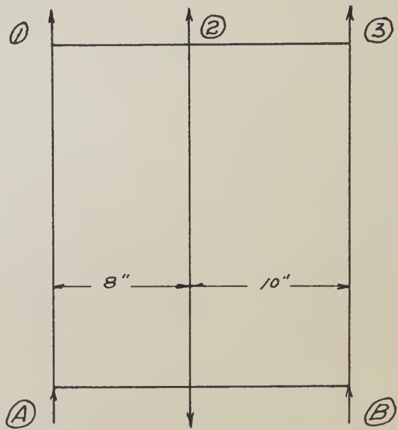


NOTE: "a" and "b" strains given in microinches for front and back of stringer.

$LOAD = E\varepsilon A$

TABLE I

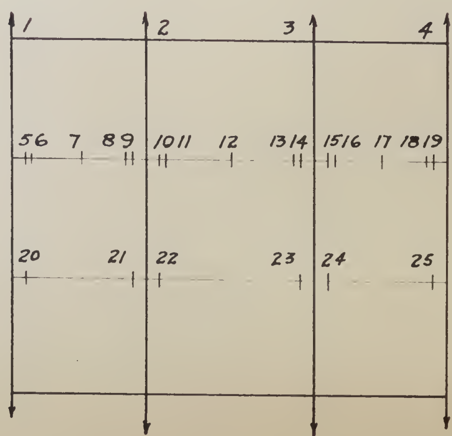
LOAD	COMPRESSION DYNAMOMETER		STRINGER (1)			STRINGER (2)			STRINGER (3)		
	(A)	(B)	1a	1b	1av	2a	2b	2av	3a	3b	3av
2000#	199	94	347	164	255	259	215	237	259	185	222
	240#	188#									



NOTE: ALL Strains given in microinches.
Dynomometer Loads obtained from calibration curve. Stringer Load = $E\varepsilon A$

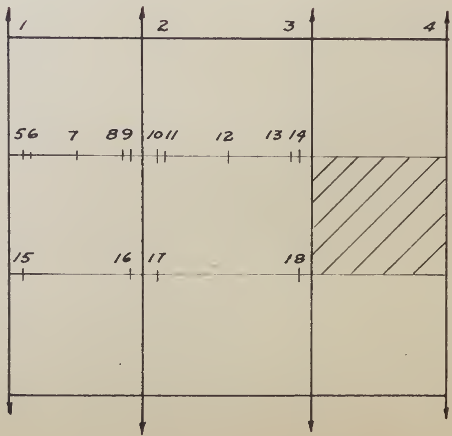
TABLE II

LOAD	STRAIN MEASUREMENTS														
	1a	1b	1	2a	2b	2	3a	3b	3	4a	4b	4	5a	5b	5
2000	153	299	226	140	383	262	101	432	266	104	352	228	59	78	68
	6a	6b	6	7a	7b	7	8a	8b	8	9a	9b	9	10a	10b	10
2000	56	73	65	38	64	51	34	73	53	33	76	54	35	72	53
	11a	11b	11	12a	12b	12	13a	13b	13	14a	14b	14	15a	15b	15
2000	30	70	50	20	53	36	30	74	52	32	80	56	34	75	54
	16a	16b	16	17a	17b	17	18a	18b	18	19a	19b	19	20a	20b	20
2000	30	72	51	26	56	41	46	77	61	49	84	67	61	67	64
	21a	21b	21	22a	22b	22	23a	23b	23	24a	24b	24	25a	25b	25
2000	71	87	79	65	84	75	26	92	59	38	83	61	25	61	43



NOTE: All Strains given in microinches

LOAD	STRAIN MEASUREMENTS														
	1a	1b	1	2a	2b	2	3a	3b	3	4a	4b	4	5a	5b	5
2000	213	234	223	179	346	262	166	399	283	210	220	215	71	54	63
	6a	6b	6	7a	7b	7	8a	8b	8	9a	9b	9	10a	10b	10
2000	72	52	62	65	35	50	75	28	52	75	26	51	71	24	48
	11a	11b	11	12a	12b	12	13a	13b	13	14a	14b	14	15a	15b	15
2000	68	22	45	57	18	38	89	55	72	100	67	84	72	82	77
	16a	16b	16	17a	17b	17	18a	18b	18						
2000	86	91	89	83	93	88	112	66	89						



NOTE: ALL strains given in microinches

TABLE IV

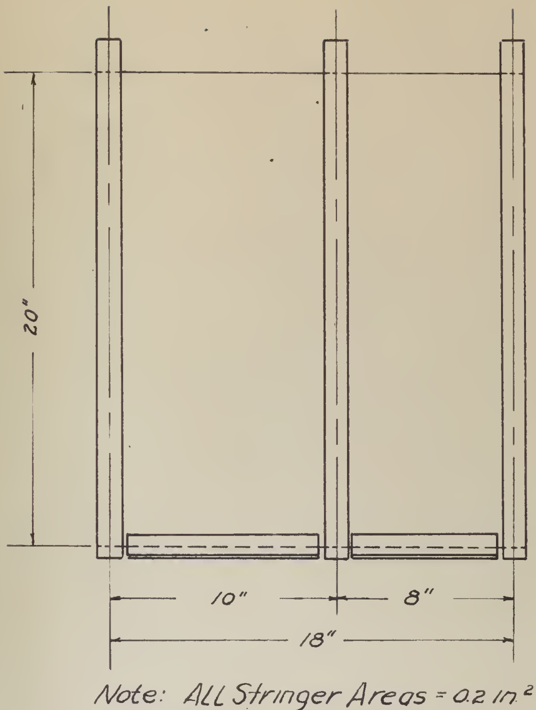
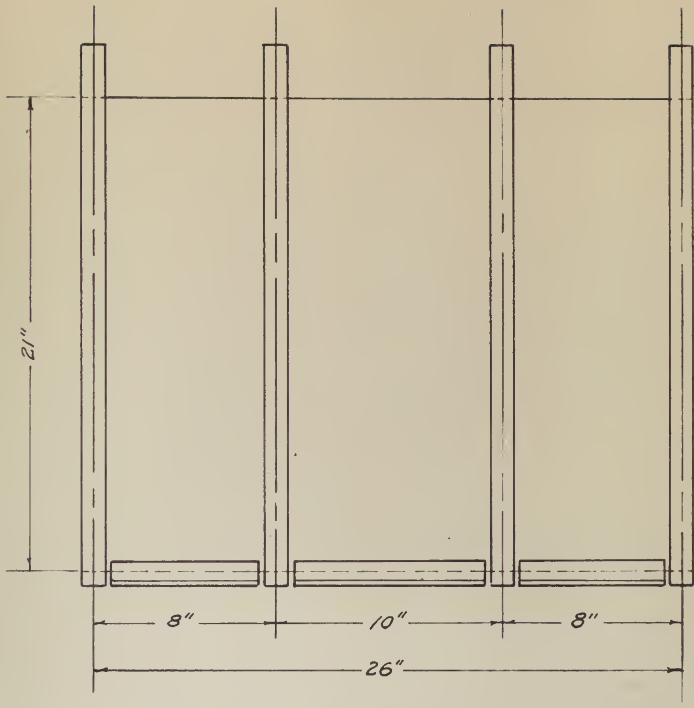
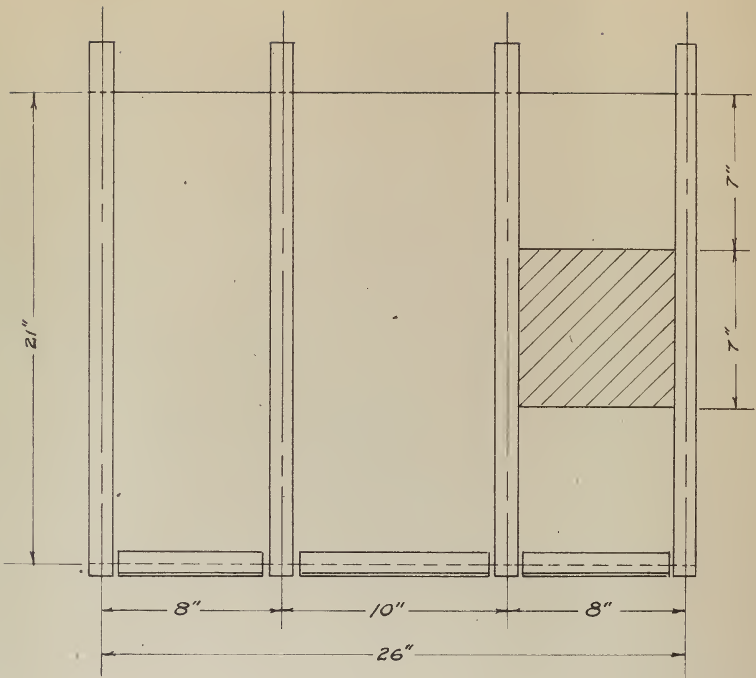


FIG. 10 THREE STRINGER MODEL



Note: All Stringer Areas = 0.189 in.^{2}

Fig. 11 Four Stringer Model



Note: All Stringer Areas = 0.189 in.^{2}

FIG. 12 FOUR STRINGER MODEL WITH CUTOUT

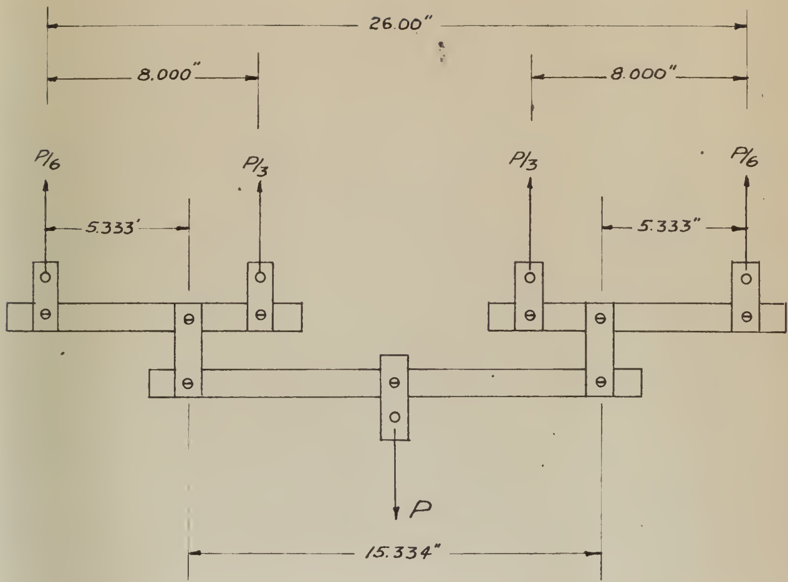


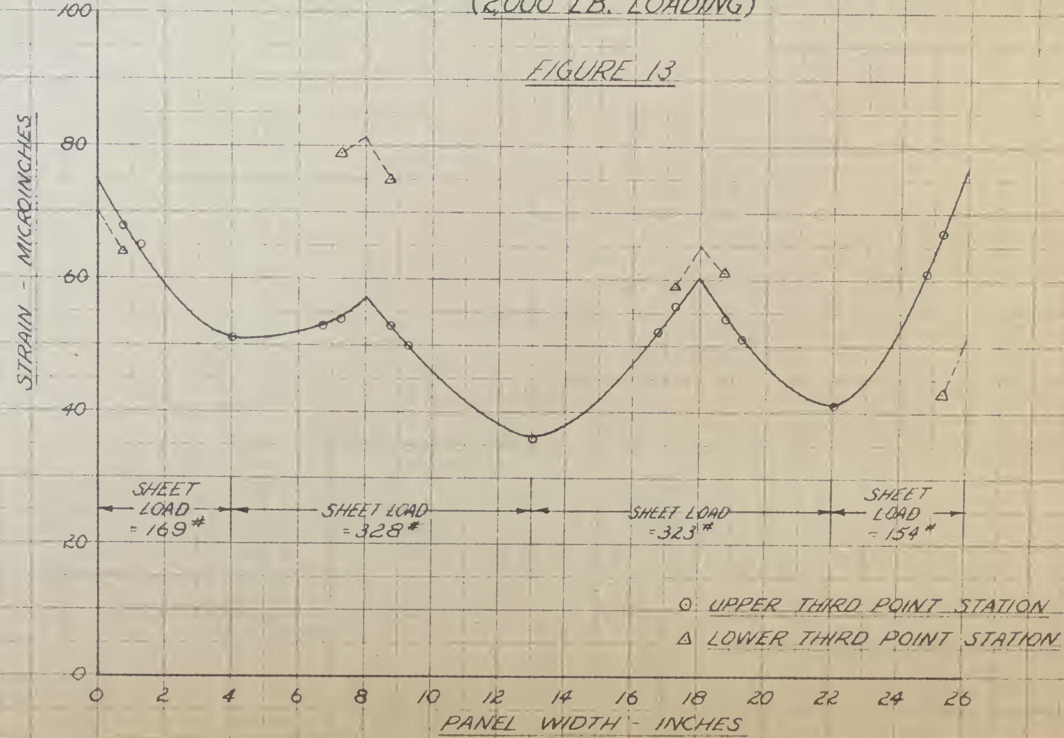
FIGURE 20 LOADING RIG FOR FOUR STRINGER MODEL

CROSS-PANEL STRAIN DISTRIBUTION

IN A FOUR STRINGER MODEL WITH NO CUTOUT

(2000 LB. LOADING)

FIGURE 13

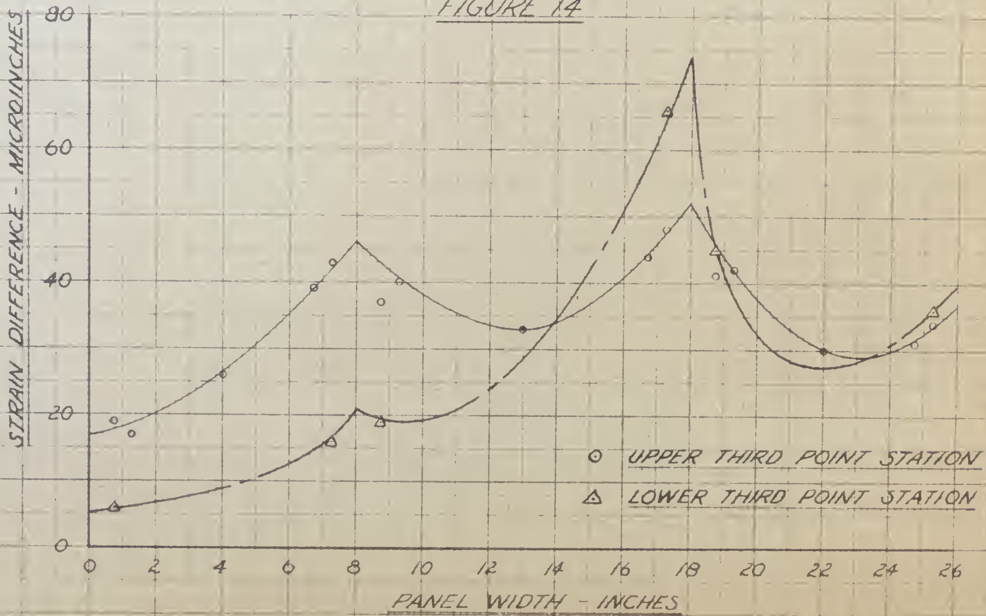


CROSS-PANEL MOMENT DISTRIBUTION

ON A FOUR STRINGER MODEL WITH NO CUTOUT

(2000 LB. LOADING)

FIGURE 14



STRESS DISTRIBUTION IN A FOUR STRINGER MODEL WITH NO CUTOUTS

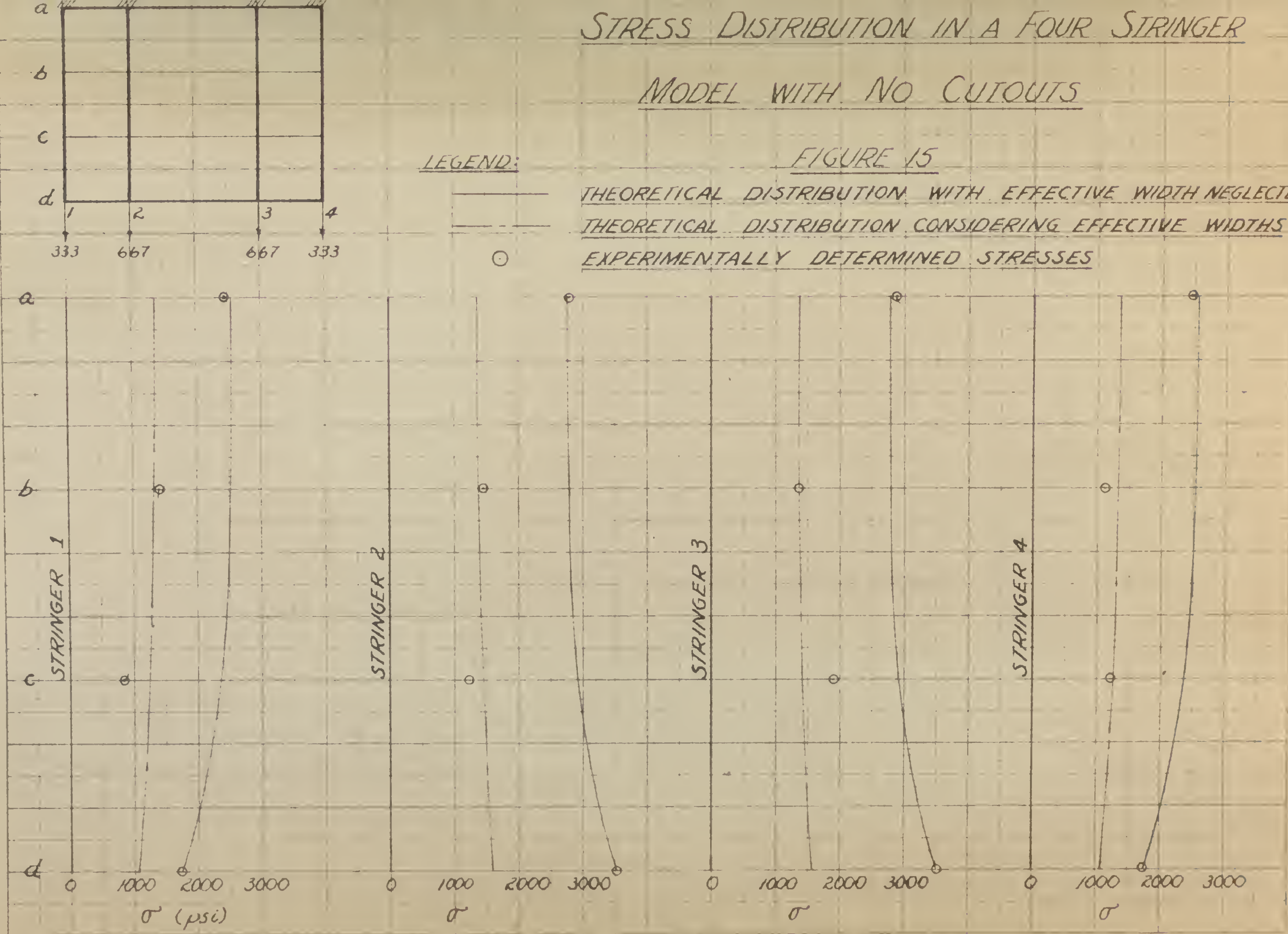
FIGURE 15

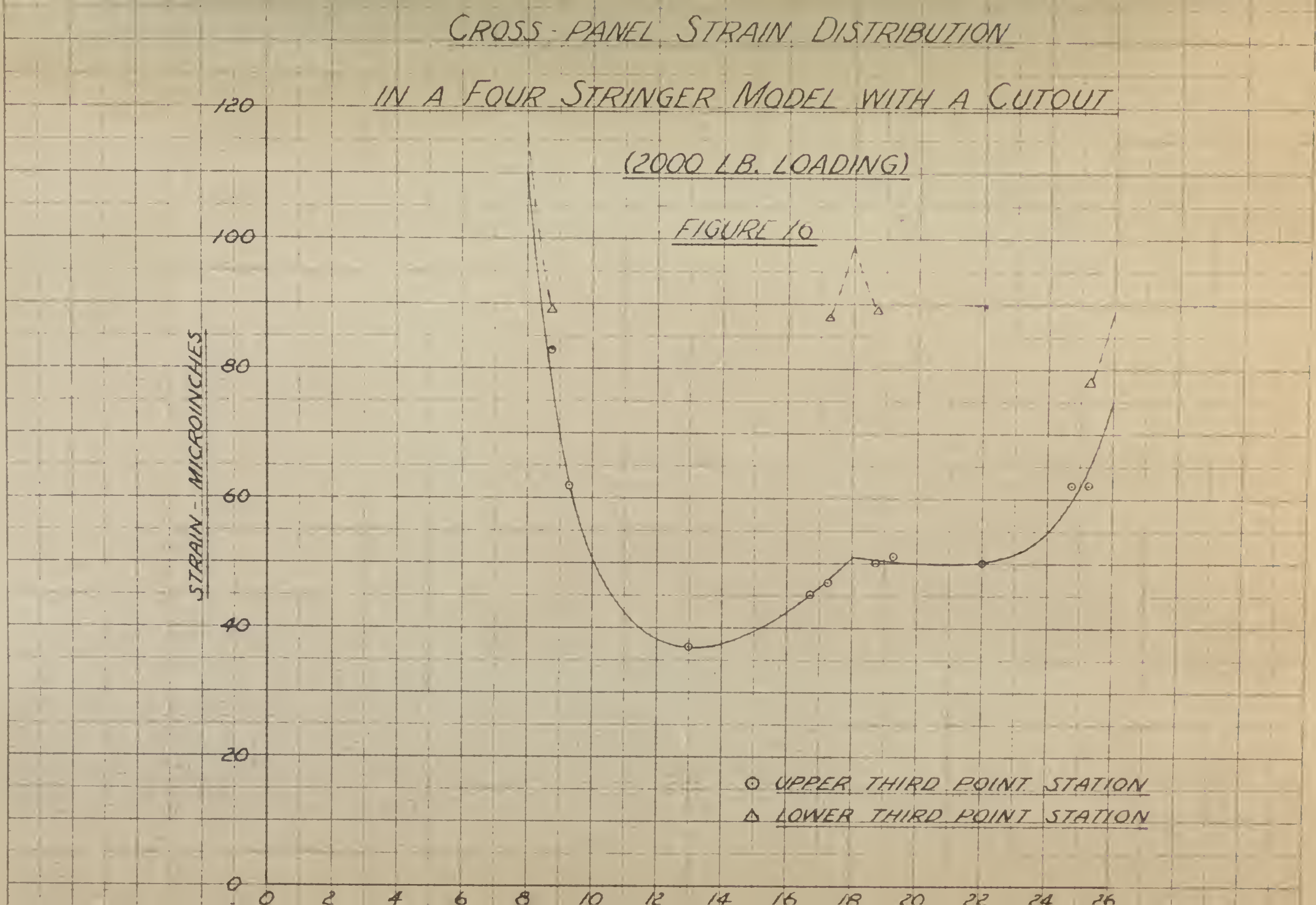
LEGEND:

THEORETICAL DISTRIBUTION WITH EFFECTIVE WIDTH NEGLECTED

THEORETICAL DISTRIBUTION CONSIDERING EFFECTIVE WIDTHS

EXPERIMENTALLY DETERMINED STRESSES



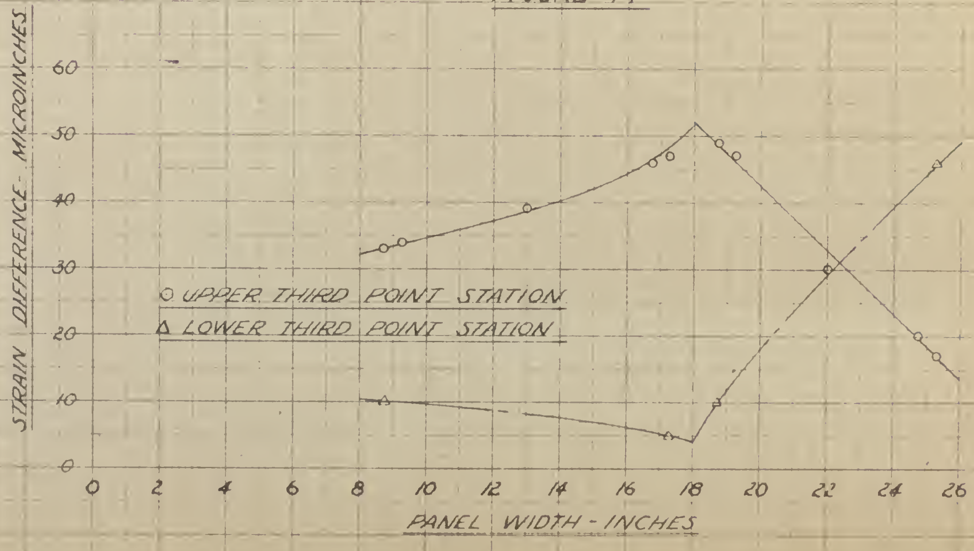

CROSS-PANEL STRAIN DISTRIBUTION
IN A FOUR STRINGER MODEL WITH A CUTOUT
(2000 LB. LOADING)
FIGURE 10
STRAIN - MICROINCHES
120
100
80
60
40
20
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26
⊙ UPPER THIRD POINT STATION
△ LOWER THIRD POINT STATION

CROSS PANEL MOMENT DISTRIBUTION

ON A FOUR STRINGER MODEL WITH NO CUTOUT

(2000 LB. LOADING)

FIGURE 17



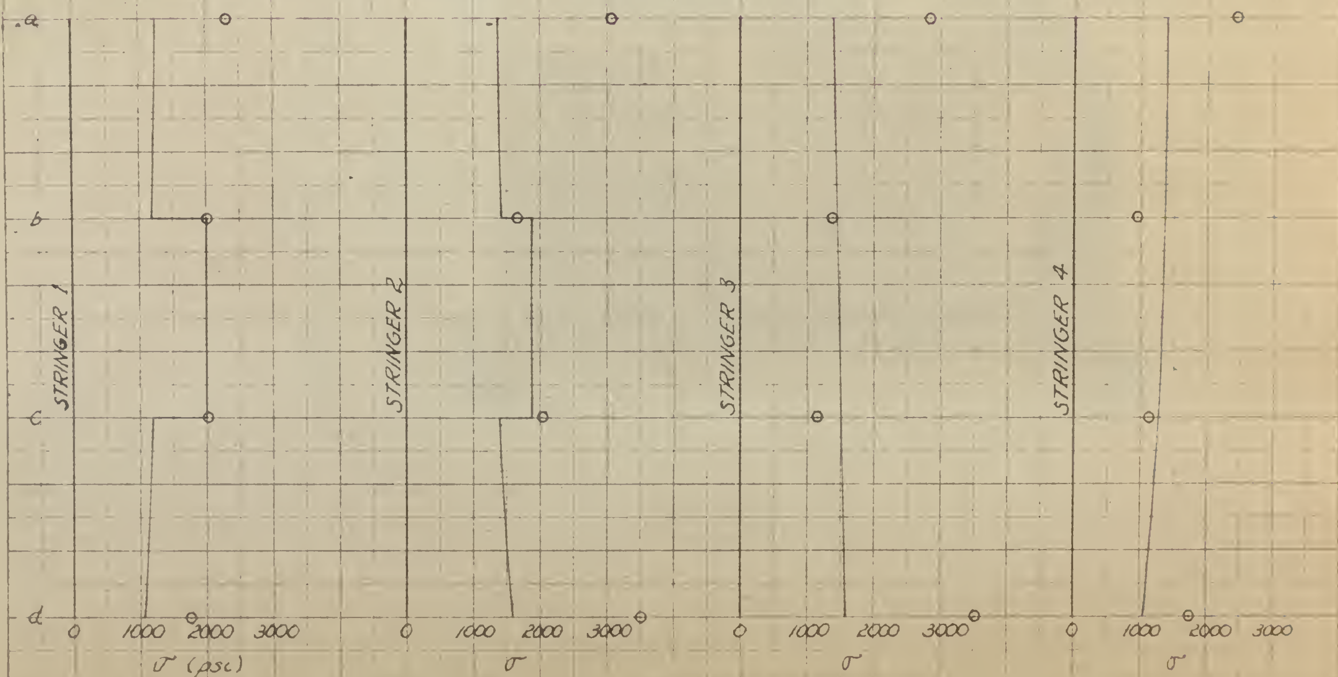
STRESS DISTRIBUTION IN A FOUR STRINGER MODEL WITH A CUTOUT

FIGURE 18

LEGEND:

—— THEORETICAL DISTRIBUTION CONSIDERING EFFECTIVE WIDTH

⊙ EXPERIMENTALLY DETERMINED STRESSES



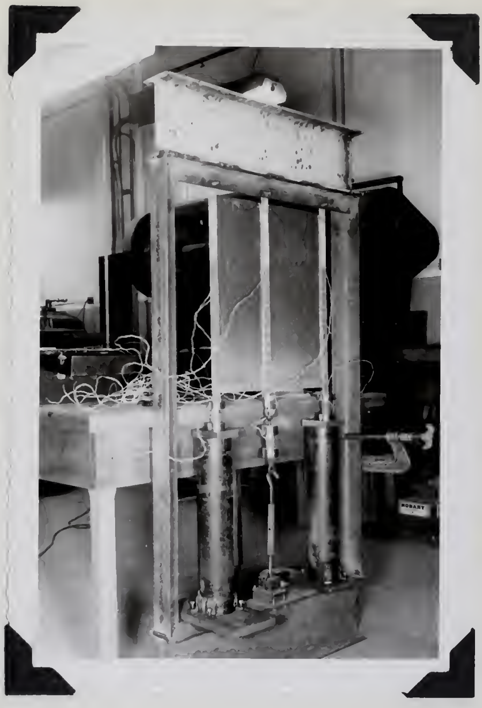


Figure 19

View showing three-stringer model mounted in rigid frame

Figure 21

View showing loading of three-stringer panel

on center stringer

Figure 21

View showing loading of three-stringer panel

on center stringer

Figure 22

View of three-stringer panel showing restraint of outer stringers and mounting of compression dynamometers

Figure 23

View of four-stringer panel with no cutout, showing the strain gage mounting pattern

Figure 24

View of the four-stringer panel with a cutout